Revealing Preferences through a Ranked-Choice Lottery.

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Abstract

In a ranked-choice lottery, applicants reveal their preferences for different characteristics of a good by ranking the options. Individuals make a trade-off between the different characteristics; and the marginal rate of substitution (MRS) between the characteristics of the top two picks reveals an individual's value for the good. The MRS indicates how much of one characteristic the individual will trade-off in order to obtain his second-pick. To determine individual preferences, this paper models applicant behavior when applying to a ranked-choice lottery, and from the model, we derive the MRS between the first and second picks. Applying the model to the USFS's Four Rivers Lottery, we show that for most applicants the MRS is approximately close to zero, suggesting that for the Four Rivers Lottery the first-pick has no effect on the second-pick.

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1. Introduction

When goods are not allocated through a pricing mechanism, individuals compete for the good through alternate mechanisms. The non-price competition allows researchers to determine individual's value for the good. For example, Barzel [4] examines the case of rationing a commodity on a first-come-first-serve basis, where individuals compete for the good through the cost of time spent in line. Examining a preference point lottery, Buschena et al. [6] shows how individuals reveal preferences for a good through preference points and time spent waiting for an elk permit. Nickerson [11] models the demand for deer hunting permits in Washington, where individuals compete for different recreational characteristics through the probability of winning.

One example of non-price competition is the ranked-choice lottery, where applicants rank a subset of options. The applicants rank each option depending on the utility generated from a pick and compete through the probability of winning each pick. Applicants can potentially win the second or subsequent ranked-options, but at a reduced probability of winning. Thus, the applicant faces a probability trade-off between his top pick and his alternative options.

By ranking his options, the applicant equates the probability trade-off he faces to his marginal rate of substitution (MRS) between the characteristics of two options. For example, the MRS between his first and second picks is the rate at which the applicant is willing to substitute characteristics of his first-pick in order to obtain his second-pick. Knowing an applicant's ranking reveals his MRS, providing a measure for the value of his second-pick. The MRS also reveals what characteristics impact the applicant's decision.

Examples of a ranked-choice lottery include the Berkeley public schooling systems, which distributed the right to attend schools with different curriculum. Some medical schools and law schools offering popular classes have switched to a ranked-choice lottery to distribute the right to enroll in those classes. New Mexico also utilizes a ranked-choice lottery when distributing elk permits with different hunting restrictions. This paper examines how individuals reveal their preferences for a good through a ranked-choice lottery, using data from the United States Forest Service's (USFS) Four Rivers Lottery. The lottery distributes permits to raft four popular sections of rivers in Idaho, each with unique characteristics.

Understanding how the characteristics of each option affects an applicant's ranked picks provides policymakers with information on the relative demand of the characteristics of goods and services. For example, in the Four Rivers Lottery, the Middle Fork of the Salmon (referred to below as the Middle Fork) provides hiking, fishing, and geothermal hot springs, but the river averages only three non-commercial and four commercial launches a day during the summer. The Selway river is more technical with several class 4 rapids but only one launch a day [14, 10]. Thus, determining preferable rafting characteristics can help policymakers determine how applicants respond to changes in policy. Such policy changes could include adjusting the number of permits available for days with favorable characteristics, or increasing the application fee to increase the probability of winning on particular days.

This paper develops a model of incentives in a ranked-choice lottery that allows for the retrieval of the relative value of a good's characteristics. From the model, we derive the MRS between the first-pick and the second-pick characteristic. Applying the model to the USFS's Four Rivers Lottery, we calculate the MRS for all applicants and reveal what first-pick characteristic an individual is willing to give-up in order to obtain the characteristics of his second-pick.

We discuss the Four River's lottery in section 2. Section 3 builds a model of an applicant's decision in applying and ranking the different options. In section 4, we estimate the model's parameters using a seemingly unrelated Poisson regression model, calculate the MRS, and discuss the results. Conclusions and policy recommendations are presented in section 5.

2. USFS's Ranked-Choice Lottery

A ranked-choice lottery distributes goods by having applicants rank their top picks. In a common ranked-choice lottery design, applicants are grouped together by their picks, and permits are awarded randomly by selecting from each group.

Applicants of the Four Rivers Lottery rank their top four picks by river and date, and the USFS distributes permits randomly selecting individuals from the group of applicants with the same first-pick. For example, one applicant chose July 4th and 5th on the Middle Fork of the Salmon River as his first and second picks and August 28th and 29th on the Snake River as his third and fourth picks. The USFS awards permits for July 4th on the Middle Fork by grouping together all applicants with that river/date as their first-pick. From that "first-pick" group, permits are awarded to randomly selected applicants. If and only if permits for that river/date remain after the "first-pick" drawing, then the USFS groups together all applicants with July 4th on the Middle Fork as their second-pick. From the "second-pick" group, the USFS randomly selects applicants. If permits still remain for that river/date option, then the process is repeated for the "third-pick" and "fourth-pick" groups. July 5th is a very popular date, and the Middle Fork is a popular river. For most dates, the number of applicants with the Middle Fork as their first-pick is larger than the number of permits. Therefore, the likelihood of anyone winning a permit for July 5th on the Middle Fork as a second-pick is extremely small.

Because individuals apply in December and January for the summer rafting season, an applicant makes his decisions based on *ex ante* expectations about the river characteristics and the probability of winning. The USFS's website provides applicants with information on daily river discharge from the past five years, the number of applicants for each river/date from the previous year, application statistics for each river from the previous three years, and the number of permits to be awarded for each river/date. Because rafting requires weathering the elements, other characteristics, such as temperature or precipitation, may also play a role in the applicant's decision. Average temperature and precipitation data are available from the National Climatic Data Center at no monetary cost, but require navigating through another website [14].

Figure 1 plots the data on discharge, temperature, precipitation, and the *ex post* probability of winning over the course of the permit season for each river. For the Main Salmon (referred to as the Main), Middle Fork, and Selway Rivers, discharge at the beginning of the season is relatively high and decreases over time, but the probability of winning at the beginning of the season is relatively low and increases over time. For the Snake River, river discharge is less variable, possibly due to the Hell's Canyon Dam, so river discharge level is likely less of a concern to individuals applying for the Snake than for individuals applying for the other three rivers.

Picking a particular river/date as the first ranked-choice comes at the cost of a reduced probability of winning other river/date picks. By selecting July 4th on the Middle Fork as his top option, the applicant has implicitly chosen the river characteristics for that river and date at the cost of his second-pick, July 5th on the Middle Fork. Thus, the applicant has made a trade-off between the characteristics (including the probability of winning) of his first and second picks. By choosing among river/date options, the individual reveals his preferences for different river characteristics through the ranking system.

3. Model of an Applicant's Decision

We assume that an applicant seeks to maximize his expected value from applying to the lottery. An individual is allowed to submit only one application, but could potentially win all four picks. The expected value from applying is comprised of the sum of the expected utility from his top four picks.[1] The expected utility of each pick depends on the probability of winning a chosen permit, and the utility from winning and using the permit. The probability of winning is defined by the river/date characteristics because more preferable characteristics induce more applicants for that option, and therefore, a lower probability of winning a permit. The utility of a pick is also based on the river/date characteristics, such as the temperature outside, the river flow, the probability of precipitation, and the day-of-the-week to launch.

Let $E[V]$ represent the expect value from a set of four ranked permit picks. An applicant maximizes $E[V]$ by choosing four rafting days with the characteristics $\mathbf{z}_{i_1}, \mathbf{z}_{i_2}, \mathbf{z}_{i_3}, \mathbf{z}_{i_4}$ such

[1] In the Four Rivers Lottery, applicants can win more than one permit provided the permits are for different rivers. For simplification, the model presented in this paper does not exclude the possibility of winning multiple permits on the same river. The model can, however, be easily extended to exclude such possibilities by including an indicator variable on the probability of winning the second, third, and fourth options.

that $i_1 \neq i_2 \neq i_3 \neq i_4$, and ranking them. The maximization problem can be written as

$$\begin{aligned}\max_{\mathbf{z}_{i_1},\mathbf{z}_{i_2},\mathbf{z}_{i_3},\mathbf{z}_{i_4}} E[V] =& EU\left(p_{i_1,1}, u\left(\mathbf{z}_{i_1}\right)\right) + EU\left(p_{i_2,2}, u\left(\mathbf{z}_{i_2}\right)\right) \\ &+ EU\left(p_{i_3,3}, u\left(\mathbf{z}_{i_3}\right)\right) + EU\left(p_{i_4,4}, u\left(\mathbf{z}_{i_4}\right)\right) \end{aligned} \tag{1}$$

where

$$p_{i,j} \equiv p(j, \mathbf{z}_i) = \frac{q_i - \sum_{l=1}^{j-1} n(l, \mathbf{z}_i)}{n(j, \mathbf{z}_i)} \cdot I_{\{q_i > \sum_{l=1}^{j-1} n(l,\mathbf{z}_i)\}}(\mathbf{z}_i). \tag{2}$$

The vector $\mathbf{z}_i$ represent the characteristics of a given river/date i, and $j = 1...4$ is a ranked-choice index. The expected utility, $EU(p_{i,j}, u(\mathbf{z}_i))$, from winning option i as the j^{th}-pick depends on the probability of winning the option as a j^{th}-pick, $p_{i,j}$, and the utility from winning that pick, $u(\mathbf{z}_i)$. The characteristics, $\mathbf{z}_i$, affect the expected utility received from a pick directly through the utility function, $u(\mathbf{z}_i)$, and indirectly through the probability of winning, p_i.[2]

The probability of winning a given river/date as a j^{th}-pick is represented by $p_{i,j}$, given by equation (2). The probability of winning i as a first-pick is the number of permits over the number of applicants choosing i as a first-pick, $p_{i,1} = \frac{q_i}{n(1,\mathbf{z}_i)}$, where q_i represent the number of permits for river/date option i. Because the USFS groups applicants by the ranking of a pick, we let $n(j, \mathbf{z}_i)$ represent the number of applicants choosing i as the j^{th}-pick. The probability of winning i as the j^{th} pick depends on the number of applicants in the prior $j-1$ drawings, because, for example, winning a permit as a second-pick requires that permits remain after the first-pick drawing. Thus, the probability of winning i as a second-pick is the number of

[2]Scrogin [13] and Boxall [5] also characterize the probability of winning as dependent upon the site chosen [3].

permits minus the number of first-pick applicants choosing i over the number of second-pick applicants choosing i, $p(2, \mathbf{z}_i) = \frac{q_i - n(1,\mathbf{z}_i)}{n(2,\mathbf{z}_i)}$. If more first-pick applicants exist than permits awarded, the second-pick drawing for i does not occur, as indicated by the indicator function, $I_{\{q_i > n(1,\mathbf{z}_i)\}}(\mathbf{z}_i)$. From the perspective of an applicant, the indicator function is unknown at the time of application. For tractability, we assume that the applicant uses a forecast of $\hat{I}_{\{q_i > n(1,\mathbf{z}_i)\}}(\mathbf{z}_i) = 1$ in his decision calculus if the predicted probability of winning the option i as a first-pick, $\frac{q_i}{\widehat{n(1,\mathbf{z}_i)}}$, is greater than or equal to 1. Thus, choosing the river/date i_2 as the second-pick instead of the first-pick reduces the probability of winning to

$$p(2, \mathbf{z}_{i_2}) = \frac{q_{i_2} - n(1, \mathbf{z}_{i_2})}{n(2, \mathbf{z}_{i_2})} \cdot \hat{I}_{\{q_{i_2} > n(1,\mathbf{z}_{i_2})\}}(\mathbf{z}_{i_2}), \tag{3}$$

which depends on the number of applicants choosing the river/date as their first or second picks, and the river/date characteristics, $\mathbf{z}_{i_2}$.

The first-order conditions from the maximization problem are

$$\frac{\partial E[V]}{\partial \mathbf{z}_{i_1}} = \frac{\partial EU}{\partial u}\frac{\partial u(\mathbf{z}_{i_1})}{\partial \mathbf{z}_{i_1}} + \frac{\partial EU}{\partial p}\frac{\partial p(1, \mathbf{z}_{i_1})}{\partial \mathbf{z}_{i_1}} = 0 \tag{4}$$

$$\frac{\partial E[V]}{\partial \mathbf{z}_{i_2}} = \frac{\partial EU}{\partial u}\frac{\partial u(\mathbf{z}_{i_2})}{\partial \mathbf{z}_{i_2}} + \frac{\partial EU}{\partial p}\frac{\partial p(2, \mathbf{z}_{i_2})}{\partial \mathbf{z}_{i_2}} = 0 \tag{5}$$

$$\frac{\partial E[V]}{\partial \mathbf{z}_{i_3}} = \frac{\partial EU}{\partial u}\frac{\partial u(\mathbf{z}_{i_3})}{\partial \mathbf{z}_{i_3}} + \frac{\partial EU}{\partial p}\frac{\partial p(3, \mathbf{z}_{i_3})}{\partial \mathbf{z}_{i_3}} = 0 \tag{6}$$

$$\frac{\partial E[V]}{\partial \mathbf{z}_{i_4}} = \frac{\partial EU}{\partial u}\frac{\partial u(\mathbf{z}_{i_4})}{\partial \mathbf{z}_{i_4}} + \frac{\partial EU}{\partial p}\frac{\partial p(1, \mathbf{z}_{i_4})}{\partial \mathbf{z}_{i_4}} = 0, \tag{7}$$

where $\frac{\partial EU}{\partial u}$ represents the marginal expected utility of a pick caused by a change in the utility from rafting, and $\frac{\partial EU}{\partial p}$ represents the marginal expected utility of a pick caused by a change in the probability of winning. Let $\frac{\partial u(\mathbf{z}_i)}{\partial \mathbf{z}_i}$ and $\frac{\partial p(j,\mathbf{z}_i)}{\partial \mathbf{z}_i}$ represent the marginal utility and marginal probability of winning caused by changes in the river/date characteristics. The MRS between choices k and j is $\frac{\partial u(\mathbf{z}_{i_k})}{\partial \mathbf{z}_{i_k}} / \frac{\partial u(\mathbf{z}_{i_j})}{\partial \mathbf{z}_{i_j}}$ for picks $k \neq j$. The model also allows for comparing the third and fourth picks. For brevity, we do not theoretically examine the MRS for these picks. Additionally, the data for the Four Rivers Lottery suggest that the probability of winning any option as a third or fourth pick is exceedingly small, implying a MRS of zero.

Assuming the expected utility function does not change across rankings implies that $\frac{\partial EU}{\partial u}$ and $\frac{\partial EU}{\partial p}$ are equal across equations (4)-(7). Then, we can divide equation (4) by (5) and rearrange to find the MRS between the first-pick and the second-pick as

$$MRS_{\mathbf{z}_{i_2}, \mathbf{z}_{i_1}} \equiv \frac{\partial u(\mathbf{z}_{i_2})}{\partial \mathbf{z}_{i_2}} / \frac{\partial u(\mathbf{z}_{i_1})}{\partial \mathbf{z}_{i_1}} = \frac{\partial p(2, \mathbf{z}_{i_2})}{\partial \mathbf{z}_{i_2}} / \frac{\partial p(1, \mathbf{z}_{i_1})}{\partial \mathbf{z}_{i_1}} \tag{8}$$

Equation (8) shows that the MRS between the first and second picks equals the ratio of the marginal probability of winning the first and second picks, at the optimal ranking.

Using equation (2), we can express the right hand side of equation (8) as

$$MRS_{\mathbf{z}_{i_2}, \mathbf{z}_{i_1}} = \frac{\frac{\partial n_{i_2,2}}{\partial \mathbf{z}_{i_2}}}{\frac{\partial n_{i_1,1}}{\partial \mathbf{z}_{i_1}}} \cdot \frac{n_{i_1,1}^2}{n_{i_2,2}^2} \cdot \frac{q_{i_2} - n_{i_2,2}\frac{\partial n_{i_2,1}}{\partial \mathbf{z}_{i_2}} + n_{i_2,1}\frac{\partial n_{i_2,2}}{\partial \mathbf{z}_{i_2}}}{q_{i_1}} \hat{I}_{\{q_{i_2} > n_{i_2,1}\}}, \tag{9}$$

where $n_{i,j} = n(j, \mathbf{z}_i)$ and $\frac{\partial n_{i,j}}{\partial \mathbf{z}_i}$ represents the change in the number of applicant caused by a change in the river/date characteristics.

The ratio given by equation (9) shows that the MRS can be completely characterized by the river/date characteristics through the number of applicants function, $n(j, \mathbf{z}_i)$. Knowing the relationship between the number of applicants and the river characteristics, we can calculate for each applicant the MRS between the first and second picks.

We calculate the MRS between the first and second picks for each characteristic in the vector of river/date characteristics, $\mathbf{z}_i$. The $MRS_{\mathbf{z}_{i_2},\mathbf{z}_{i_1}}$ is the rate at which the applicant is willing to substitute characteristics of his first-pick for his second-pick characteristics. A zero $MRS_{\mathbf{z}_{i_2},\mathbf{z}_{i_1}}$ implies that changes in the first-pick have no affect on the individual's second-pick; and a zero $MRS_{\mathbf{z}_{i_2},\mathbf{z}_{i_1}}$ occurs when the probability of winning the second-pick is effectively zero.

In the next section, we develop an empirical model for estimating the $MRS_{\mathbf{z}_{i_2},\mathbf{z}_{i_1}}$ from the Four Rivers Lottery data and report results.

4. Data and Regression Model

Table 1 provides summary statistics for the data used in this analysis. The mean number of applicants varies substantially. The Middle Fork averages the highest number of applicants at 103.65 per river day and the Snake averages the lowest number of applicants at 9.48, which is an order of magnitude lower than the Middle Fork. The Middle Fork also has the highest standard deviation for the number of applicants even though it has the highest mean number of applicants.

Our data include observations on river discharge, precipitation, temperature, number of permits awarded, and day-of-the-week for launching. The United States Geological Survey provides data on river discharge averaged over 1977-2006, which is included to account for the predicted flow of the river [2]. The National Climatic Data Center provides data on temperature and precipitation averaged over 1971-2000, which are included to account for predicted weather characteristics [1]. The number of permits range from 0 to 8, depending on the river.

4.1. Estimation of the number of applicants

We use a seemingly unrelated Poisson regression model to account for the discrete number of applicants, and the joint decision of choosing option i as the first or second pick. The probability of observing $n_{i,1}$ number of first-pick applicants and $n_{i,2}$ number of second-pick applicants for option i is

$$P(N_{i,1} = n_{i,1}, N_{i,2} = n_{i,2} | \theta_{i,1}, \theta_{i,2}, \xi) = exp(\xi - \theta_{i,1} - \theta_{i,2}) \sum_{j=0}^{min(n_{i,1}, n_{i,2})} \frac{\xi^j}{j!} \frac{(\theta_{i,1} - \xi)^{(n_{i,1}-j)}}{(n_{i,1} - j)!} \frac{(\theta_{i,2} - \xi)^{(n_{i,2}-j)}}{(n_{i,2} - j)!} \quad (10)$$

where $N_{i,1}$ represents the random variable of the number of applicants choosing i as their first-pick, and each observation, $n_{i,1}$, is drawn from a univariate Poisson distribution with parameter $\theta_{i,1} = \lambda_{i,1} + \xi$, such that $n_{i,1} = 0, 1, 2, ...$ Additionally, $\theta_{i,1}$ depends upon the data of river/date characteristics for option i. Similarly, each observation for the number of second-pick applicants, $n_{i,2}$, is drawn from a univariate Poisson with parameters $\theta_{i,2} = \lambda_{i,2} + \xi$, such that $n_{i,2} = 0, 1, 2, ...$ The correlation between the two options is represented by ξ. If $\xi \neq 0$, the model gains efficiency over an equation-by-equation Poisson model by accounting for potential correlation between the disturbance processes associated with the first and second picks [9]. The Poisson distribution restricts the mean and variance for the number of applicants to be equal.

To ensure the mean parameter, $\theta_{i,k}$, is non-negative, we assume the expected number of applicants choosing option i as a first-pick or second-pick is

$$E\left[n_{i,1}|\mathbf{z}_{i_1}\right] = \theta_{i,1} = e^{\mathbf{z}'_{i_1}\boldsymbol{\beta_1}} \tag{11}$$

$$E\left[n_{i,2}|\mathbf{z}_{i_2}\right] = \theta_{i,2} = e^{\mathbf{z}'_{i_2}\boldsymbol{\beta_2}}. \tag{12}$$

To examine the relationship between the number of applicants, $n_{i,1}$ and $n_{i,2}$, and the river/date characteristics, $\mathbf{z}_{i_1}$ and $\mathbf{z}_{i_2}$, we estimate the parameter vectors $\boldsymbol{\beta_1}$ and $\boldsymbol{\beta_2}$ from the Poisson model using a maximum likelihood technique [7]. The log-likelihood function is

$$\begin{aligned} ln(L) = \sum_{i=1}^{M} & \left[\xi - e^{Z'_{1i}\boldsymbol{\beta_1}} - e^{Z'_{2i}\boldsymbol{\beta_2}}\right] \\ & +ln\left(\sum_{j=0}^{min(n_{i,1},n_{i,2})} \frac{\xi^j}{j!}\frac{(e^{Z'_{i,1}\boldsymbol{\beta_1}} - \xi)^{(n_{i,1}-j)}}{(n_{i,1}-j)!}\frac{(e^{Z'_{i,2}\boldsymbol{\beta_2}} - \xi)^{(n_{i,2}-j)}}{(n_{i,2}-j)!}\right). \end{aligned} \tag{13}$$

Three different models were examined for the relationship between the number of applicants and the river/date characteristics. The first model includes the regressors temperature, precipitation, discharge, day-of-the-week and a river dummy variable. The model accounts for a possible non-linear relationship by including a quadratic term for temperature, precipitation, and discharge. To examine the relationship for specific rivers, the second model adds interaction terms between the river dummy variable and the day-of-the-week to launch. We also interact the river dummy variable with previous years' river discharge to account for different rafting preferences for each river and because information on past discharge is more easily accessible to rafters. Further evidence that the amount of discharge impacts the number of applicants is illustrated in figure 1, where a relationship between discharge and the probability of winning is more distinct than a relationship between temperature and the probability of winning, for the Main, Middle Fork, and Selway Rivers. Finally, the third model considers all possible interactions between rivers and the characteristics, including temperature and precipitation.

4.2. Results and discussion

The estimated coefficients of the three models are presented in tables 2a and 2b, and used to calculate the MRS between the first and second option. Parameters for the number of first-pick applicants are presented in the odd numbered columns; and the parameters for number of second-pick applicants are presented in the even numbered columns.

In table 2a, the estimated coefficient for *Permits* captures the effects of differences in the number of permits. The positive coefficient implies that a larger number of permits is associated with more fist-pick and second-pick applicants for a given river/date. Two potentially opposing factors influence how *Permits* affects the number of applicants: congestion and the probability of winning. Increases to the number of permits increases the probability of winning because there are more permits to win. This is likely to increase the number of applicants for a given river/date with a higher probability of winning.[3] Yet, an increase in the number of permits also increases congestion, which may decrease the value from rafting on these wilderness rivers, and therefore may induce a decrease in the number of applicants. Thus, a positive estimated coefficient for *Permits* implies that the congestion factor has less of an impact on the number of applicants than the probability of winning factor.

[3]At a minimum, the probability of winning would remain the same, implying that the probability could not decrease as a consequence of an increase in the number of permits [12].

To control for unobserved differences across rivers, dummy variables were included for the Snake, Middle Fork, and the Selway, leaving the Main as the base case. All models suggest the Middle Fork has the highest number of applicants, and models 1 and 2 suggest that the Snake averages the lowest number of applicants.

For the discharge variable in table 2a, the estimated coefficients suggest that increases in river discharge increase the number of applicants at a decreasing rate for all for rivers except the Snake River. For the Middle Fork, the number of applicants peaks when river discharge is about 3.71 thousand cubic feet per second, which occurs around July 15th. Similarly, for the Main applicants peak when discharge is 8.34 thousand cubic feed per second, which occurs around June 20th. From figure 1, we observed the Snake river had the least distinct relationship between discharge and the probability of winning, indicating that discharge is less of a factor for individuals applying for a Snake River permit. Figure 2 illustrates the estimated relationship between discharge and the number of applicants by plotting discharge against the predicted and actual number of applicants. For the Snake, figure 2(b) illustrates that no relationship between discharge and the number of applicants exists, but for the Main, Middle Fork, and Selway a more distinct relationship exists. Thus, the $Discharge$ coefficients suggests that river discharge affects what day an individual chooses on the Main, Middle Fork, and Selway, but not on the Snake River. Again, discharge variation on the Snake is small because of the Hell's Canyon dam that is upriver controlling the river discharge flow.

Table 2b shows the estimated coefficients for the temperature and precipitation variables. In most of the models temperature is not statistically significant, suggesting that temperature has little influence on the number of applicants. Additionally, the results for precipitation are not statistically significant for most models, suggesting that precipitation has little influence on the number of applicants. The estimated coefficients in model 1 are statistically significant, but with signs opposite our hypothesis that more precipitation decreases the number of applicants at an increasing rate. Thus, being the most restrictive model with the largest AIC, the precipitation results in model 1 suggest an omitted variables bias. The influence of temperature and precipitation on the number of applicants may be small for several reasons. Other factors such as discharge and launch day-of-the-week might play a larger role in how individuals apply. Also, the USFS does not provide information on temperature and precipitation on the application website as it does with discharge and popular days-of-the-week, making the prediction of temperature and precipitation more difficult.

For brevity, day-of-the-week estimates are not included in tables 2a and 2b, but table 3 shows the impact from an expected change in choosing a different day of the week to launch. The dummy variable, z_k, for each day-of-the-week is transformed using the method proposed by Kennedy [8]. For the Main, Middle Fork, and Selway rivers, early in the week (i.e. Sunday, Monday, and Tuesday) has a higher number of applicants, suggesting that early launch days are preferable. For the Snake River, a Friday launch day averages the highest number of applicants compared to any other day-of-the-week.

4.3. Calculation of the MRS

The estimated coefficients from model 3 in tables 2a and 2b are used in conjunction with equations (11) and (12) to calculate the marginal effects $\widehat{\frac{\partial n(j,\mathbf{z}_i)}{\partial \mathbf{z}_i}}$ and predictions from the Poisson model for $\widehat{n(1,\cdot)}$ and $\widehat{n(2,\cdot)}$. We calculate the MRS in equation (8), which reveals an individual's preferences for the river/date characteristics.

As is evident in equation (8), the number of applicants in the first-pick drawing affects whether the second-pick drawing occurs. The individual does not know with certainty that the second-pick drawings will or will not occur, and must choose based on a prediction of the probability of a drawing occurring for his second choice. For most river/date options, the predicted probability of a second-pick drawing is approximately zero. For simplicity, we set the indicator function in equation (2), $\hat{I}_{\{q_i > \widehat{n(1,\mathbf{z}_i)}\}}(\mathbf{z}_i)$, equal to zero when the predicted number of first-pick applicants is greater than the number of permits. Predictions for the number of applicants are calculated using model 3.

From the model 3 predictions, we determined that only 49 applicants out of $16,257$ chose a second-pick where the predicted number of first-pick applicants was less than the number of permits. The remaining $16,208$ chose a second-pick with a predicted probability of winning that is effectively zero, implying the MRS is zero. The zero MRS suggests that given the current lottery, marginal changes in characteristics of an applicant's first-pick does not affect his second-pick.

For the 49 applicants where the probability of winning the second-pick is non-zero, we calculate the MRS for each of the river characteristics used in model 3. In table 4, we present the mean MRS for each characteristic by grouping the applicants by their first-pick river and then again by their second-pick river. The MRS's are interpreted as giving up a first-pick characteristics, $\mathbf{z}_{i_1}$, in order to obtain the bundle of characteristics in their second-pick, $\mathbf{z}_{i_2}$.

Averaging the MRS by the first-pick river reveals how individuals value the characteristics of their top choice. For example, the mean $MRS_{\mathbf{z}_{i_2},\mathbf{z}_{i_1}}$ for the Middle Fork shows that the average individual choosing the Middle Fork as a first-pick is willing to give-up river discharge, temperature, and precipitation but not permits in order to obtain their second-pick. The average individual choosing the Main, Snake, and Selway rivers as a first-pick is willing to give-up additional permits.

Averaging the MRS by the second-pick river, we compare how individuals value their second-pick. For example, the MRS for the discharge characteristic is the largest for Middle Fork applicants, implying that applicants choosing the Middle Fork as a second-pick gain a greater value from their second-pick than applicants choosing the Main, because they are willing to give-up more discharge in their first-pick. Furthermore, the MRS for discharge suggests that applicants choosing the Main as a second-pick gain a greater value from their second-pick than applicants choosing the Snake. The MRS's for the permits and precipitation characteristics provide similar results.

5. Conclusions and Future Research

When goods are not allocated through a pricing mechanism, alternate models must be used to determine the value of the good. This paper identifies another dimension in which individuals compete for a non-market good. In a ranked-choice lottery, applicants compete through the probability of winning more preferable characteristics of the good. The ranking of options reveals how much uncertainty an individual is willing to accept in order to obtain his second-pick, and the trade-off between the first and second picks reveals an individual's MRS between his first and second picks and his preference for the characteristics of the good.

In this paper, we modeled an applicant's decision in applying to a ranked-choice lottery. We examine preferences by calculating the MRS between the first and second picks. MRS's for rafting permits from the Four River Lottery are calculated by estimating the relationship between the number of applicants and the river/date characteristics, predicting the number of applicants for each river/date option, and applying the estimated parameters and predictions to the calculation of the MRS. The MRS reveals what characteristics the individual is willing to give-up in his first-pick in order to obtain the characteristics of his second-pick.

Individual preferences are examined through the calculation of the MRS. For most applicants the MRS is zero, implying that marginal changes to their first-picks do not affect their second-picks. This result comes from the fact that most applicants choose a second-pick that effectively has a zero probability of winning. Thus, for most applicants the second-pick does not matter. For individuals with a non-zero MRS, the results indicate that the average applicant choosing the Middle Fork as a second-pick is willing to give up river discharge, temperature, and precipitation in order to obtain their second-pick. Furthermore, applicants choosing the Middle Fork as a second-pick gain a greater value from their second-pick than individuals choosing the Main or Snake as a second-pick.

Examining how applicants make their decision, we can assist policymakers in what river characteristics matter to applicants. For the Four Rivers Lottery, discharge has the largest impact on an individual's decision to Middle Fork and Selway Rivers. Temperature and precipitation play a less critical roll in an individual's decision. Typically, the number of permits and day-of-the-week do play a significant roll in an individual's decision.

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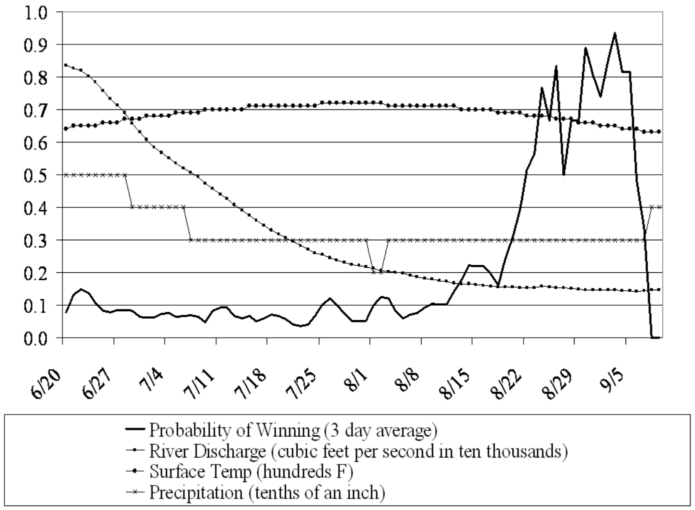
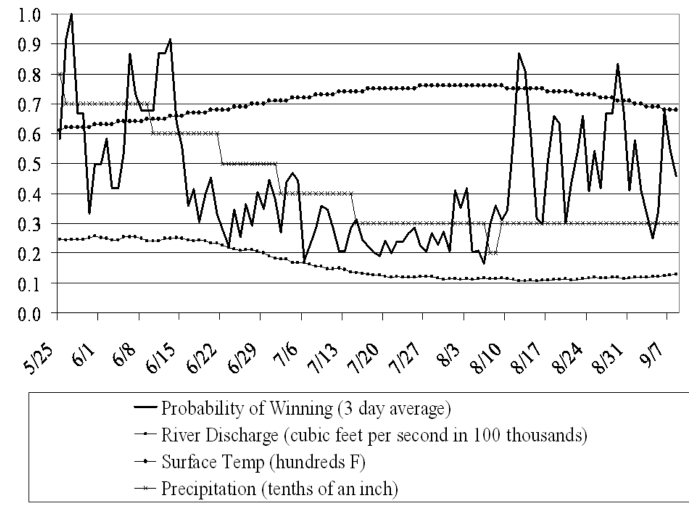
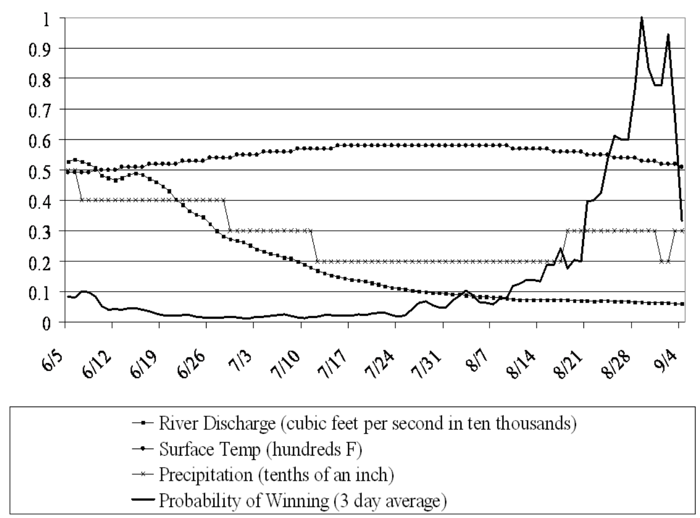
Figure 1: River Characteristics by Date for the (a)Main, (b)Snake, (c)Middle Fork and (d)Selway Rivers.



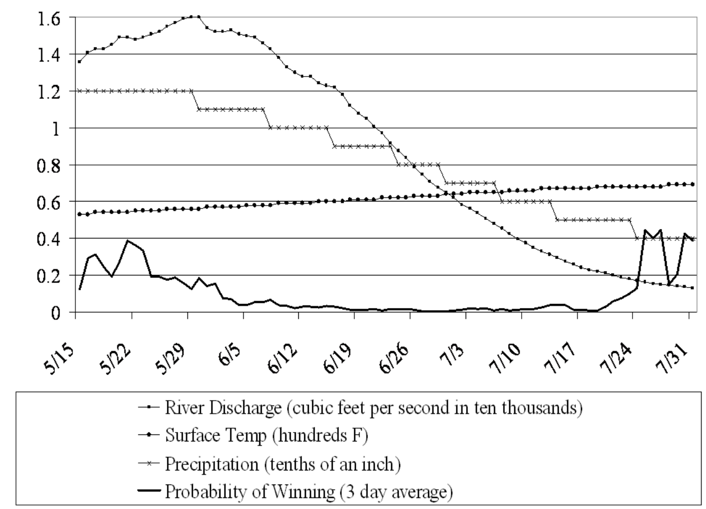
(a) Main Salmon



(b) Snake



(c) Middle Fork



(d) Selway

Table 1: Descriptive Statistics by River

Main - Variable	Min	Max	Mean	St. Dev.	N
Applicants (first-pick)	0	118	42.68	32.07	80
Applicants (second-pick)	1	127	49.03	32.57	80
Permits	1	5	3.76	0.66	160
Temp. (°F)	63	72	68.83	2.52	160
Precipitation (hundredths of an inch)	2	5	3.30	0.70	160
Discharge (thousands of cubic feet per second)	1.42	8.36	3.27	2.12	160

Snake - Variable	Min	Max	Mean	St. Dev.	N
Applicants (first-pick)	0	46	9.48	9.31	109
Applicants (second-pick)	0	59	12.24	9.23	109
Snake - Permits	0	3	2.59	0.86	218
Temp.	61	76	70.72	4.47	218
Precipitation	2	8	4.23	1.56	218
Discharge	10.7	25.8	16.43	5.60	218

Middle Fork - Variable	Min	Max	Mean	St. Dev.	N
Applicants (first-pick)	1	453	103.65	98.93	99
Applicants (second-pick)	3	347	85.85	76.97	99
Permits	1	8	3.72	1.16	198
Temp.	47	58	54.47	3.349	198
Precipitation	2	5	2.95	0.88	198
Discharge	.613	5.59	2.29	1.74	198

Selway - Variable	Min	Max	Mean	St. Dev.	N
Applicants (first-pick)	0	95	24.15	21.79	62
Applicants (second-pick)	1	77	20.40	17.00	62
Permits	0	2	1	0.18	124
Temp.	53	69	61	5.23	124
Precipitation	4	12	8.69	2.93	124
Discharge	1.31	16	9.67	5.55	124

Table 2a: Simultaneous Poisson Model

	Model 1		Model 2[a]		Model 3[a]	
	1st Pick	2nd Pick	1st Pick	2nd Pick	1st Pick	2nd Pick
	(1)	(2)	(3)	(4)	(5)	(6)
Permits	0.152***	0.126**	0.092**	0.061*	0.085***	0.053
	(0.058)	(0.052)	(0.040)	(0.036)	(0.039)	(0.036)
Middle Fork	11.949***	9.061***	3.282***	2.55***	57.27	92.13
(MF)	(0.878)	(0.634)	(0.570)	(0.465)	(108.451)	(70.337)
Snake	-9.584***	-7.339***	-6.241***	-4.22***	115.4	57.25
	(0.954)	(0.654)	(1.476)	(1.386)	(119.815)	(83.622)
Selway	1.06***	0.262	-0.689	-0.71	10.61	37.36
	(0.396)	(0.345)	(0.865)	(0.685)	(128.304)	(99.338)
Discharge	0.732***	0.541***	0.538***	0.493***	0.634***	0.413**
	(0.088)	(0.067)	(0.140)	(0.108)	(0.239)	(0.203)
Discharge^2	-0.013***	-0.009***	-0.016	-0.02	-0.038	-0.013
	(0.003)	(0.002)	(0.017)	(0.013)	(0.036)	(0.030)
Dis. - MF	-	-	2.263***	2.11***	2.537***	2.502***
	-	-	(0.224)	(0.192)	(0.308)	(0.260)
Dis.^{2} - MF	-	-	-0.343***	-0.319***	-0.389***	-0.389***
	-	-	(0.037)	(0.033)	(0.049)	(0.042)
Dis. - Snake	-	-	-0.108	-0.213	-0.097**	-0.047
	-	-	(0.216)	(0.193)	(0.355)	(0.279)
Dis.^{2} - Snake	-	-	0.01	0.017	0.031	0.007
	-	-	(0.018)	(0.013)	(0.037)	(0.030)
Dis. - Selway	-	-	0.457**	0.363**	-0.707	-0.316
	-	-	(0.185)	(0.150)	(0.457)	(0.522)
Dis.^{2} - Selway	-	-	-0.028	-0.021	0.038	0.005
	-	-	(0.019)	(0.014)	(0.039)	(0.034)
Correlation	1.281 ***		-0.056		-0.336	
	(0.123)		(0.413)		(0.482)	
AIC	12176.461		6865.47		6567.897	
BIC	12326.646		7243.21		7054.863	

[a]For brevity, estimated coefficients for day-of-week categorical variables are omitted. Model 1 includes no river interaction variables. Model 2 interacts river with discharge and day-of-the-week. Model 3 includes the interactions of Model 2 and interacts river with temperature and precipitation. Bootstrapped standard errors are reported in the parenthesis. Significance levels: $*$:10% $**$:5% $***$:1%.

Table 2b: Simultaneous Poisson Model

	Model 1		Model 2[a]		Model 3[a]	
	1st Pick	2nd Pick	1st Pick	2nd Pick	1st Pick	2nd Pick
	(1)	(2)	(3)	(4)	(5)	(6)
Temp	0.0545	0.187	0.094	0.253	0.733	2.657
	(0.164)	(0.140)	(0.211)	(0.171)	(3.064)	(2.002)
$Temp^2$	0.005***	0.003**	0.002	0	-0.002	-0.017
	(0.002)	(0.001)	(0.002)	(0.001)	(0.022)	(0.014)
Temp - Snake	-	-	-	-	-3.285	-1.682
	-	-	-	-	(3.459)	(2.411)
$Temp^2$ - Snake	-	-	-	-	0.022	0.012
	-	-	-	-	(0.025)	(0.017)
Temp - MF	-	-	-	-	-1.711	-2.434
	-	-	-	-	(3.178)	(2.080)
$Temp^2$ - MF	-	-	-	-	0.014	0.017
	-	-	-	-	(0.023)	(0.015)
Temp - Selway	-	-	-	-	-0.068	-0.513
	-	-	-	-	(4.010)	(3.231)
$Temp^2$ - Selway	-	-	-	-	-0.003	-0.001
	-	-	-	-	(0.031)	(0.026)
Precip	1.343***	1.077***	-0.22	-0.278**	-0.081	-0.384
	(0.170)	(0.168)	(0.159)	(0.139)	(1.166)	(0.927)
$Precip^2$	-0.084***	-0.066***	0.019	0.025**	0.052	0.072
	(0.011)	(0.008)	(0.016)	(0.013)	(0.170)	(0.134)
Precip - Snake	-	-	-	-	0.395	-0.452
	-	-	-	-	(1.623)	(1.154)
$Precip^2$ - Snake	-	-	-	-	-0.166	0.025
	-	-	-	-	(0.217)	(0.159)
Precip - MF	-	-	-	-	-0.889	-1.245
	-	-	-	-	(1.275)	(1.069)
$Precip^2$ - MF	-	-	-	-	0.081	0.15
	-	-	-	-	(0.186)	(0.157)
Precip - Selway	-	-	-	-	2.287	1.022
	-	-	-	-	(1.440)	(1.158)
$Precip^2$ - Selway	-	-	-	-	-0.177	-0.115
	-	-	-	-	(0.179)	(0.139)
Correlation	1.281 ***		-0.056		-0.336	
	(0.123)		(0.413)		(0.482)	
AIC	12176.461		6865.47		6567.897	
BIC	12326.646		7243.21		7054.863	

[a]For brevity, estimated coefficients for day-of-week categorical variables are omitted. Model 1 includes no river interaction variables. Model 2 interacts river with discharge and day-of-the-week. Model 3 includes the interactions of Model 2 and interacts river with temperature and precipitation. Bootstrapped standard errors are reported in the parenthesis. Significance levels: ∗:10% ∗∗:5% ∗∗∗:1%.

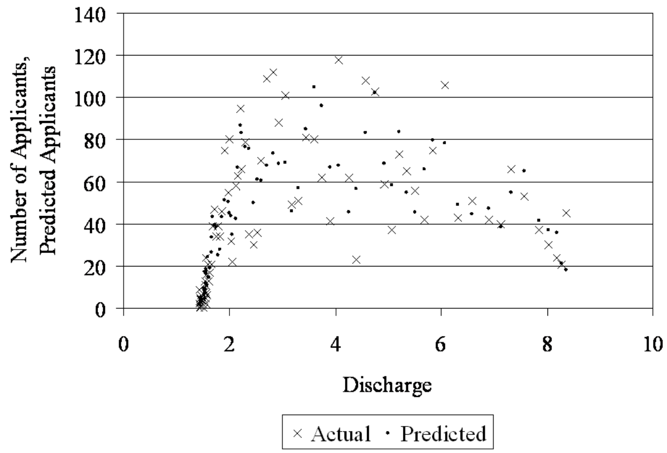
Table 3: The impact of switching from a Saturday launch date on the number of applicants for each river.

Model 1	**Sun.**	**Mon.**	**Tue.**	**Wed.**	**Thu.**	**Fri.**	**Sat.**
	42.74%	45.83%	18.74%	18.99%	11.17%	27.54%	0%

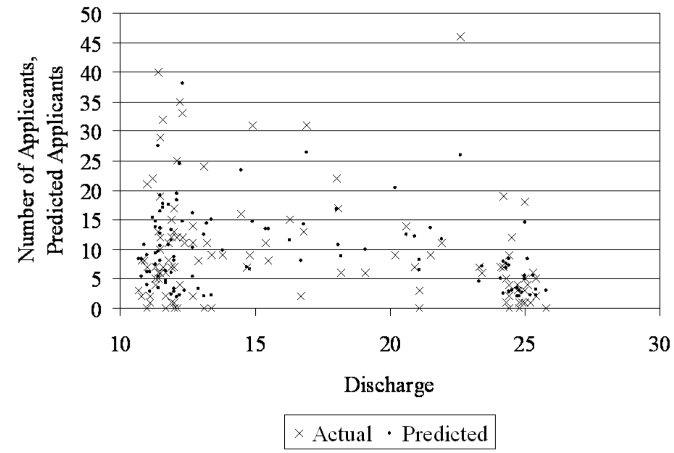
Model 2	**Sun.**	**Mon.**	**Tue.**	**Wed.**	**Thu.**	**Fri.**	**Sat.**
Main	47.31%	36.85%	31.15%	15.54%	-7.19%	9.74%	0%
Snake	-11.52%	13.38%	-20.80%	-27.14%	2.55%	68.58%	0%
Middle Fork	22.63%	51.54%	39.52%	61.29%	28.76%	12.03%	0%
Selway	9.85%	32.51%	8.22%	-0.91%	-22.50%	-30.62%	0%

Model 3	**Sun.**	**Mon.**	**Tue.**	**Wed.**	**Thu.**	**Fri.**	**Sat.**
Main	10.32%	16.21%	-0.23%	-30.33%	-40.54%	-7.02%	0%
Snake	3.03%	8.76%	-37.70%	-40.58%	-0.97%	81.44%	0%
Middle Fork	57.76%	60.25%	45.45%	50.57%	39.04%	23.02%	0%
Selway	7.44%	13.24%	19.48%	6.11%	-28.66%	-13.71%	0%

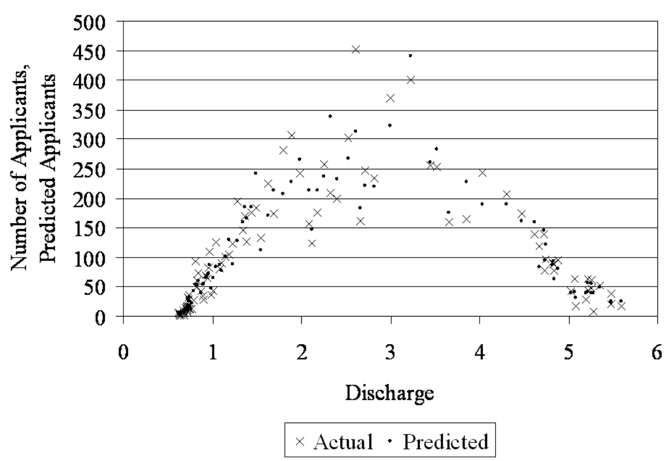
Figure 2: River Characteristics by Discharge (a)Main, (b)Snake, (c)Middle Fork and (d)Selway Rivers.



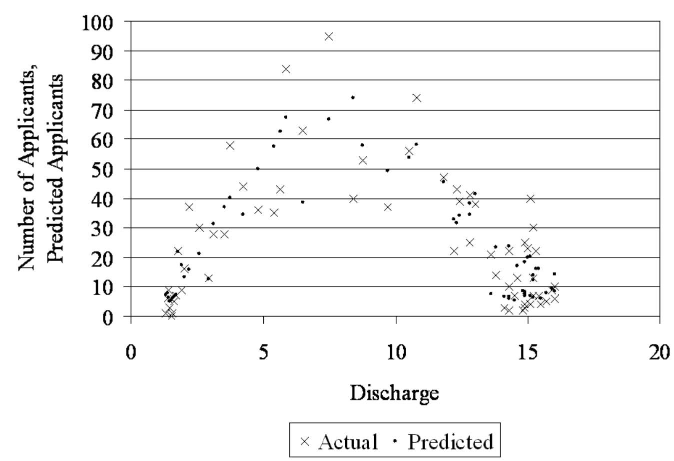
(a) Main Salmon



(b) Snake



(c) Middle Fork



(d) Selway

Table 4: Mean $MRS_{\mathbf{z}_{i_2},\mathbf{z}_{i_1}}$ by river characteristic.[a]

MRS averaged over applicants with the same first-pick river				
	Permits	**Discharge**	**Temp.**	**Precip.**
Main	0.66	-0.89	-52.53	-2.18
Snake	0.20	-1.06	2.41	50.29
Middle Fork	-28.55	36335.74	156.21	131.10
Selway	1.00	-132.19	-1476.24	23.73
MRS averaged over applicants with the same second-pick river				
	Permits	**Discharge**	**Temp.**	**Precip.**
Main	-4.38	-58.38	-1117.13	-242.438
Snake	-50.03	-342.29	582.843	50.29
Middle Fork	18.27	3972.18	-47.86	-669.45
Selway[b]	n/a	n/a	n/a	n/a

[a]MRS's are calculated from eqn. (8) for each individual that chose a second-pick with a non-zero predicted probability of occurring. Using the results in tables 2a and 2b, we calculated the marginal effects evaluated at the characteristic levels of each individual's first and second pick, and use the marginal effects to calculate each individual's MRS. Presented in this table is the mean MRS averaged over the first-pick river, using model 3 estimates.
[b]No dates on the Selway were predicted to have a second-pick drawing.